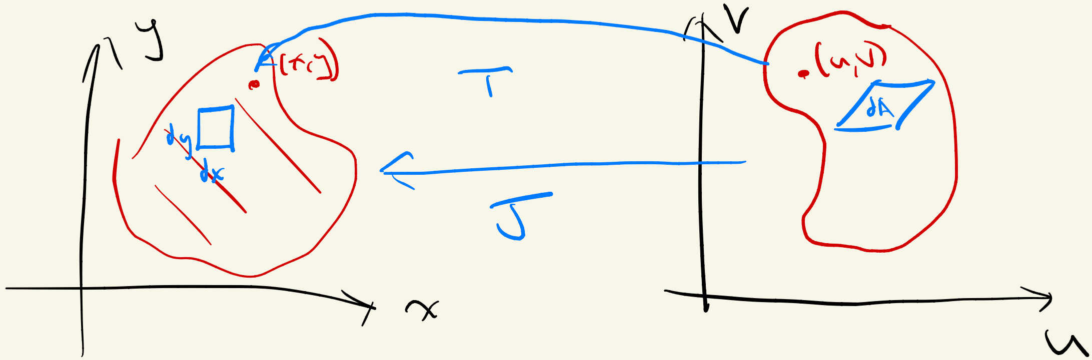


- For T:

$$(u,v) \xmapsto{T} (x,y)$$

input output

$$\begin{bmatrix} x \\ y \end{bmatrix} = T\left(\begin{bmatrix} u \\ v \end{bmatrix}\right)$$

- The Jacobian J is the "derivative":

$$(du, dv) \xmapsto{J} (dx, dy)$$

changes in x, y

Jacobian determinant

$\det(J)$

tells us how area changes

output = J · input

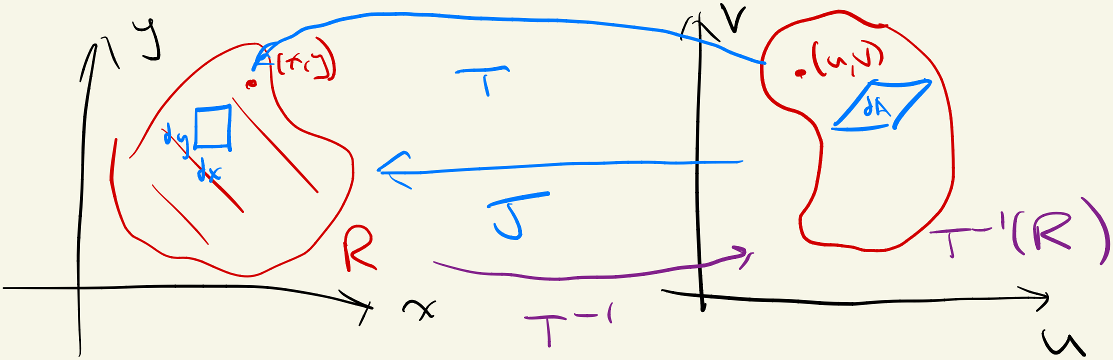
$$\begin{bmatrix} dx \\ dy \end{bmatrix} = J \cdot \begin{bmatrix} du \\ dv \end{bmatrix}$$

chain rule: $= \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} \begin{bmatrix} du \\ dv \end{bmatrix}$

This is two equations:

$$dx = \frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv$$

$$dy = \frac{\partial y}{\partial u} du + \frac{\partial y}{\partial v} dv$$



How does integration respond?

1D:

$$\int_a^b f(x)\,dx = \int_{T^{-1}(a)}^{T^{-1}(b)} f(T(u))\,\underbrace{\underbrace{T'(u)}_{\det(J)}\,du}_{dx}$$

2D:

$$\iint_R f(x,y)\,dx\,dy = \iint_{T^{-1}(R)} f(T(u,v))\,\underbrace{\det(J)\,du\,dv}_{dA}$$

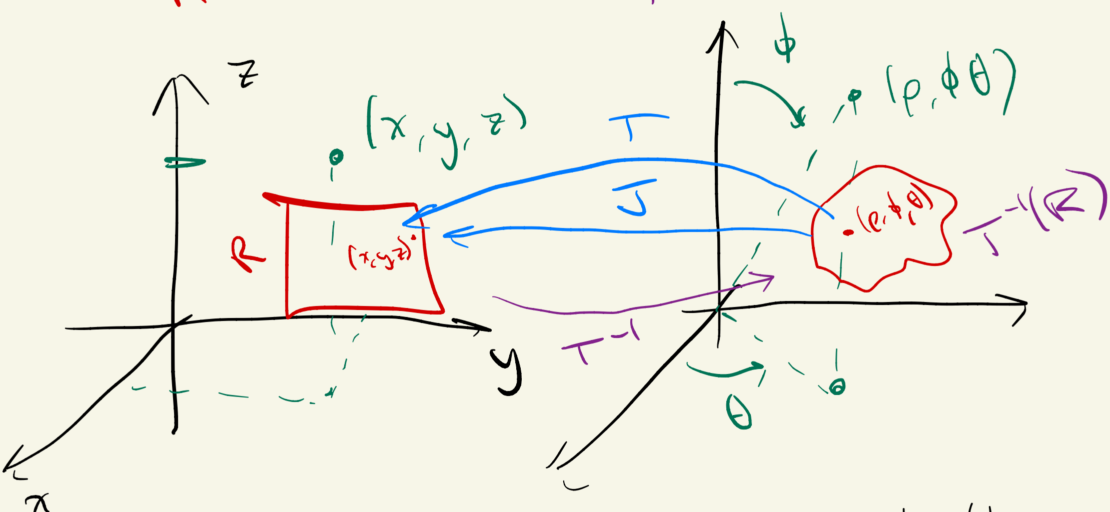
"T transforms (u,v) coord to (x,y) coord

$$\Rightarrow \begin{bmatrix} dx \\ dy \end{bmatrix} = J \cdot \begin{bmatrix} du \\ dv \end{bmatrix}$$

$$\Rightarrow dx\,dy = \det J\,du\,dv$$ "

Note that this feels "backwards"!
When we change coordinates between spherical to rectangular, we write

$$\iiint_R f(x,y,z)\,dx\,dy\,dz = \iiint_{T^{-1}(R)} f(T(\rho,\phi,\theta))\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$



To avoid confusion: write explicitly

$$T : (\rho,\phi,\theta) \longrightarrow (x,y,z)$$

$$\Rightarrow \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = T\begin{pmatrix} \rho \\ \phi \\ \theta \end{pmatrix}$$

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = J\begin{bmatrix} d\rho \\ d\phi \\ d\theta \end{bmatrix}$$